Rheology of Coal. V. Some Factors Affecting the Strength of Coal

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Introduction

The authors already attempted in Part IV[1] an absolute expression of the strength of coal from measuring the stress required to fracture, f, and the duration of the stress, t, and hence the strength was discussed by the values of "fatigue modulus" and molal activation energy for fracture. Further results are described in this paper with statistical treatment on the factors affecting the strength of coal. The stress at rupture and activation energy, which should have been compared for samples with constant analytical composition and also at same duration of stress, are again given for several typical cases using the statistical correlation equations of f upon t and ash concentration.

Further Experiments on the Fracture of Coal

Table 1 shows the values of f, t covering mainly the range of 2 to 0.1 second, Young's modulus from the vibrational method, E_d, Young's modulus from the static (bending) method, E_s, as well as ash content in weight % and in volume %, ϕ. The measurements were carried cut for f and t by the previously described apparatus to measure the breaking strength at varying durations 5 of load[1]. The dynamic Young's modulus was obtained by the acoustic method measuring the resonant frequency of the rectangular specimen at audiofrequencies[2]. The static Young's modulus was calculated by measuring the deformation of the specimen end by reflection of light from the mirror cemented on one of the ends, at bending of the rectangular specimen when the stress is applied to the center of the length and the specimen is supported with two wedge-shaped metals at near positions to the ends. The experimental apparatus used includes that for the fracture measurement and is schematically shown in Fig. 1. The equation by which the static modulus of elasticity, E_s, was calculated is as follows:

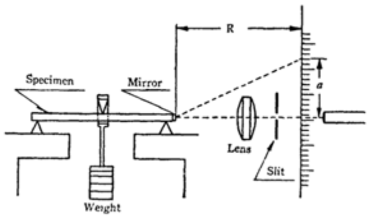

Fig. 1.—Apparatus measuring the Static Young's Modulus

$$E_s=\frac{3Wl^2R}{2b\tau^3a},$$

where W is the weight, l the span length of specimen, b the breadth, τ the thickness, R the distance between scale and the mirror cemented on one end of the specimen, and a the deviation of the reflected light spot.

The W/a value was taken from the "elastic" strain-stress relation, i.e., the linear relationship which is obtained from the total strain minus plastic deformation remaining after unloading.

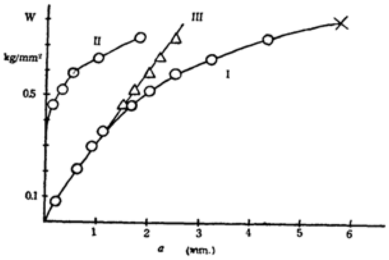

Fig. 2.—An Example of Static Measurement
I, Total Strain
II, Plastic Deformation
III, "Elastic" Strain (I−II)
The crossing denotes the breaking point.

Fig. 2 shows an example of the static experiments for a sample from American coal.

The samples of coal examined were selected from the species which had been discussed in the preceding articles of this study, i.e., American bituminous (strongly-caking) coal, Takashima bituminous (caking) coal, Toro bituminous (weakly-caking) coal and Shosaku anthracite. The descriptions of the samples in general properties are given in Part II or III[3,4]. All specimens were cut from blocks in rectangular bar with the accuracy of 1% in each dimension and parallel to the bedding plane. The measurements were carried out after putting the samples in the dessicator of 76% relative humidity for 6 months to avoid the influences owing to the moisture in the sample.

Table 1

Strength, Young's Moduli and Properties of Coal

Sample	t (sec.)	f (kg./mm.²)	$E_d\times10^{-10}$ (dyne/cm.²)	$E_s\times10^{-10}$ (dyne/cm.²)	Ash Content (dry basis) In weight (%)	In volume (ϕ)
American	0.1	1.16	7.35	1.19	55.16	42.05
(Bituminous,	0.1	0.09	1.89	0.12	4.45	2.72
strongly-	0.1	0.19	2.84	0.19	5.07	3.07
caking)	0.1	0.16	3.34	0·23	2.87	1.74
	0.1	0.81	13.4	1.20	52.02	39.41
	0.5	0.53	3.94	0.55	9.90	6.28
	0.5	0.20	4.14	0.23	5.03	3.08
	0.5	0.66	14.6	1.15	80.05	70.65
	1	0.09	2.65	0.15	1.77	0.87
	2	0.69	7.86	0.92	87.40	80.61

1) K. Inouye, This Bulletin, **26**, 200 (1953).
2) K. Inouye, *J. Colloid Sci.*, **6**, 190 (1951).
3) K. Inouye, This Bulletin, **26**, 84 (1953).
4) K. Inouye, This Bulletin, **26**, 157 (1953).

Takashima	0.1	0.15	4.59	0.17	3.40	2.27
(Bituminous,	0.1	0.37	3.40	0.59	20.59	13.46
caking)	0.1	0.22	2.91	0.20	1.61	0.97
	0.1	0.40	3.98	0.42	4.02	2.44
	0.1	0.50	3.08	0.42	4.75	2.89
	0.1	0.40	4.19	0.43	8.59	5.34
	0.5	0.11	2.27	0.10	2.80	1.69
	1	0.29	2.96	0.28	3.70	2.25
	60	1.03	4.74	0.62	2.12	1.28
Toro	0.1	0.12	2.27	0.13	6.94	4.28
(Bituminous,	0.1	1.09	4.52	0.65	6.05	3.72
weakly-caking)	0.1	0.46	2.97	0.37	7.91	4.90
	0.1	1.05	6.83	0.65	10.43	6.53
	0.5	0.68	3.71	0.44	10.35	6.48
	0.5	0.16	2.37	0.18	1.14	0.69
	1	0.93	4.49	0.51	6.45	3.97
	2	1.40	3.94	0.73	9.04	5.63
	2	0.28	3.44	0.19	7.48	4.63
	10	0.31	3.55	0.14	5.80	3.56
Shosaku	0.1	1.14	7.90	0.91	8.80	5.47
(Anthracite)	0.1	0.92	9.35	0.90	8.51	5.30
	0.1	0.28	7.70	0.36	7.83	4.85
	0.1	1.00	11.5	0.74	6.68	4.11
	0.1	0.41	6.87	0.44	6.31	3.88
	0.5	1.21	9.83	0.80	4.94	3.02
	0.5	0.57	5.98	0.57	8.98	5.59
	2	0.85	7.49	0.67	5.64	3.46

Strength and Young's Modulus

Some correlations have been expected between the Young's modulus which expresses the bond strength in the solid and the breaking strength. Tables 2 and 3 show the statistical expressions of the correlation. On the whole, samples with high elastic constant show high strength, assuming the duration of load, t, is always constant; in these calculations the Young's modulus is assumed to be independent of the resonant frequency at which the measurement was carried out, and also the dependence of f on t is ignored. In other words, the errors shown in the table contain the errors owing to these possible dependencies.

It would, however, appear that a strong correlation of strength exists upon the static Young's modulus rather than the dynamic Young's modulus. This fact will be naturally accepted as the fracture experiments have been carried out at the range of t mainly of 2 to 0.1 second, longer duration comparing with the rate of deformation by the acoustic method when the resonant frequency was approximately 5,000 c/s.

Reading the figures in Tables 2 and 3, E values are given in 1×10^{10} dyne/cm.^{2} unit, f in 1×10^{8} dyne/cm.^{2} unit.

Table 2
Strength and Static Young's Modulus

Sample	Correlation Coefficient	Regression Line of E_s upon f	Standard Error	Test of Correlation
American	0.94	$E_s=0.05+1.27\ f$	0.16	<0.01
Takashima	0.70	$E_s=0.16+0.48\ f$	0.12	<0.05
Toro	0.84	$E_s=0.13+0.44\ f$	0.12	<0.01
Shosaku	0.94	$E_s=0.24+0.60\ f$	0.07	<0.01

Table 3
Strength and Dynamic Young's Modulus

Sample	Correlation Coefficient	Regression Line of E_d upon f	Standard Error	Test of Correlation
American	0.69	$E_d=2.35+8.92\ f$	3.11	<0.05
Takashima	0.36	$E_d=2.99+1.28\ f$	0.79	<0.3
Toro	0.71	$E_d=2.53+2.12\ f$	0.86	<0.05
Shosaku	0.63	$E_d=5.80+3.42\ f$	1.27	<0.1

Dynamic Young's Modulus and Static Young's Modulus

The dynamically obtained elastic constant is remarkably larger than the static modulus of elasticity; the experimental result is understood from the general concept in rheology that the static deformations force the material deform accompanied with some flows. This characteristic is recognized especially in American bituminous coal, with 35% volatile matter in mean value, having remarkable difference between two series of elastic moduli.

The statistical calculations in Table 4 suggest high correlation between E_d and E_s. Comparing also E_d and E_s in Table 1 for each specimen, it may be significant to note the facts, (1) for each coal, E_s variations owing to every affecting factor are less than E_d variations, (2) in the range of low ash content, E_d/E_s ratio is markedly large, probably because of the influence of viscous flow, as low ash content means high content of bituminous matter which acts as the plasticizer (refer Part I and II of this study), and (3) E_d/E_s ratio is large especially for the bituminous coals. These points appear to support the proposed concept of coal structure from the rheological standpoint; one of the authors' (K. I.) previous experiments has shown that, in strongly-caking coals, ash is the centre of solvation of organic coal molecules, just like a "filler" in the plastics, and the molecules with weaker intermolecular forces, surrounding such a rigid micelle structure of solvated molecules, act as a "plasticizer", especially when the whole system is carbonized, during which a softening of the system at say 400~500°C. is characteristically to be accomplished. The weakness of intermolecular forces of "plasticizer" molecules makes them distinguished in being easily extracted by solvents. Therefore, for example, the so-called bitumen (extract by benzene up to 300°C. at 50 atmosphere) content, B, is a function of ash concentration in volume, ϕ, as $\phi = a/B^2$[2]. It would be natural to interpret the wide difference between E_d and E_s for low ash content samples as a phenomenon based on remarkable viscous flow by weakly combined molecules. However, the authors consider that more detailed measurements by a refined apparatus would be necessary for further discussions.

Table 4

Young's Moduli from Dynamic and Static Methods

Sample	Correlation Coefficient	Regression Line of E_d upon E_s	Standard Error	Test of Correlation
American	0.90	$E_d=1.12+8.60\ E_s$	1.89	<0.01
Takashima	0.53	$E_d=2.66+2.68\ E_s$	0.72	<0.1
Toro	0.75	$E_d=2.12+4.26\ E_s$	1.01	<0.05
Shosaku	0.52	$E_d=5.32+4.48\ E_s$	1.40	<0.2

Correlation of Strength on Ash Content

As one of the authors (K. I.) has proposed in the preceding paper[2], Young's modulus is a linear function of ash concentration (dry basis) in volume %, ϕ, for each species, and hence the properties of the coal are characterized by the solvation factor, K, of molecules around ash and also the modulus at $\phi=0$, E_0. If the theory is valid, and if the fact is accepted that the strength is also a linear function of E (especially the static Young's modulus, as described above), the strength is expected to be in simple relationship with ϕ.

Table 5

Strength and Ash Content

Sample	Range of ϕ (%)	Correlation Coefficient	Regression Line of f upon ϕ	Standard Error	Test of Correlation
American	0.9— 3.1	0.73	$f=0.045+0.037\ \phi$	0.03	<0.2
	6.3—13.5	0.53	$f=0.35\ +0.028\ \phi$	0.13	<0.3
	42.0—87.0	0.53	$f=0.045+0.014\ \phi$	0.35	<0.1
	0.9—87.0	0.80	$f=0.085+0.016\ \phi$	0.29	<0.01
Takashima	1.0— 2.0	−0.62	$f=1.32\ -0.665\ \phi$	0.21	<0.2
	2.0— 4.0	0.57	$f=0.001+0.14\ \phi$	0.11	~ 0.2
	4.0—22.3	0.75	$f=0.065+0.063\ \phi$	0.30	<0.1
	1.0—22.3	0.46	$f=0.36\ +0.03\ \phi$	0.33	<0.05
Yubetsu	8.8—16.9	0.63	$f=0.68\ +0.048\ \phi$	0.13	<0.3
	19.2—30.3	−0.66	$f=3.93\ -0.090\ \phi$	0.39	<0.2
	8.8—30.3	0.26	$f=1.26\ +0.024\ \phi$	0.49	<0.5
Shosaku	3.0— 5.6	−0.45	$f=1.93\ -0.24\ \phi$	0.39	<0.1

Table 5 shows the f—ϕ relationship for three present samples (Toro coal was omitted for low correlation) as well as Yubetsu coal, a producer gas coal. (Data for Yubetsu coal are shown in Table 1 of Part IV.) The dependence of t upon f is ignored in these calculations. The calculations were carried out for several ranges of ϕ of each coal; the correlation differs with the ϕ ranges. The better correlations in accordance with the division have led the authors to consider the properties of samples for each divided range of ϕ. One example will be discussed in the next paper for Takashima coal.

Correlation of Strength upon Ash and Time

The strength is the function of not only ϕ, but is correlated with both ϕ and t. A formulation between $1/f$, ϕ (or $1/\phi$) was tried using also data reported in Part IV of the study. The multiple correlation coefficients suggest, however, that a better formula could be found; low correlation may be due to the assumption that $1/f$ is in a linear relationship with ϕ (or $1/\phi$) and also with logt.

For an example of the calculation, a formula for American coal, which showed best correlation within the samples examined, is given in Table 6.

Table 6

Strength, Time and Ash Content

Sample	Multiple Correlation Coefficient	Regression Line of $1/f$	Standard Error
American	0.87	$1/f=1.09+11.2\frac{1}{\phi}+0.445\log t$	1.45

The molal activation energy for fracture, U, is obtained according to Equations (5) and (6) of Part IV of the study, i.e.,

$$U=\frac{2.3KTN}{fm},$$

where, K is the Boltzman constant, T the absolute temperature, N the Avogadro's number and m is a "fatigue modulus", i.e., the inclination of the linear relationship between $1/f$ and $\log t$. In Table 2 of the preceding paper, U values were given however for samples with varying ash concentration. It was therefore apparently difficult to compare the values for the optional conditions.

Using the formula in Table 6 between $1/f$, $1/\phi$ and $\log t$ for American coal, U values can be obtained and compared at assumed constant ash concentration and at the same values of t. Table 7 shows the U values for several typical cases.

Table 7

Strength and Molal Activation Energy for Fracture at $t=0.1$, 1 and 10 sec and $\phi=1$, 5 and 10%

Sample	t (sec)	$\phi=1$ f (kg/mm²)	$\phi=1$ U (Kcal/mol)	$\phi=5$ f (kg/mm²)	$\phi=5$ U (Kcal/mol)	$\phi=10$ f (kg/mm²)	$\phi=10$ U (Kcal/mol)
American	0.1	0.084	35.2	0.35	8.6	0.57	5.3
	1	0.081	36.4	0.30	9.9	0.45	6.6
	10	0.079	38.0	0.27	11.2	0.38	7.9

The temperature was assumed as 15°C, the average room temperature during the experiments. The American bitumious coal is remarkably weak at low ash contents and at the same time needs high activation energy for fracture. The dependence of strength on duration of load, t, is also obtained. Such a treatment would enable to describe the strength of the coal in more definite expression than routine methods used in coal industry.

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